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The Potential of Director Theory for Modelling Blood Flow in the Cardiovascular System

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Introduction

We are looking at using director theory (also known as Cosserat theory) as an alternative to classical 1D models for arterial modelling of the human cardiovascular system with the intent of balancing accuracy with low computational cost.

Methods

The director theory is a hierarchical, theory where the velocity is approximated by a series of vectors that depend only on the co-axial direction and time, multiplied by shape functions that depend only on the cross-section. Director theory can retain more of the structure of complex geometries than classical 1D modelling. Preliminary discussion and comparison of director theory to classical 1D models are outlined in Robertson and Sequeira [1].

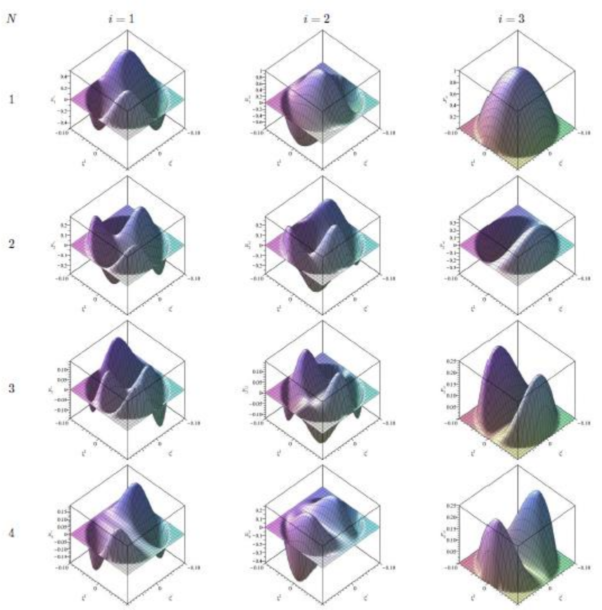

Fig 1: Plots of the shape functions $\lambda^i_N(\zeta^1, \zeta^2)$ for order N=1,2,3,4 and basis vector component i=1,2,3, with ζ^1 along the bottom right axis, ζ^2 along the bottom left axis and λ^i_N along the vertical axis.

Results & Discussion

The first results we obtained by applying director theory to fluid, following the approach of Caulk and Naghdi [2], were for Poiseuille flow and steady swirling flow in a straight pipe of constant radius.

We now begin to look at curvature, following the approach of Green *et al* [3]. Fig 1 shows plots of the shape functions for a toroidally curved pipe of circular cross-section.

Conclusion

We show in this abstract the potential for director theory for modelling the cardiovascular system without resorting to the over simplification required by classical 1D modelling.

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